Physics

Unit 12: Special Relativity

1. Define inertial reference frame, proper time, dilated time, proper length, contracted length, relativistic momentum, nonrelativistic momentum,
2. Know the relativity postulates and their consequences.
3. An astronaut travels at 1×10^8 m/s for 24 hours as measured by ground control. What is the time as measured by the astronaut?
4. An alien flies by a football game at 0.90c and measures the time it takes to kick a field goal as 0.50 s. What is the proper time for the kick?
5. A meter stick is measured to be 50 cm long. How fast must the meter stick be traveling?
6. What is the relativistic momentum of an electron traveling at 0.99c?
7. A car is 500 kg at rest. What is the increase in its energy when it is traveling at 0.90c?
8. How much energy will be released when 2 kg of pencil is converted to energy?
9. What is the ratio of relativistic kinetic energy to classical kinetic energy for a 500 kg car traveling at 0.90c?
10. The Enterprise moves at 0.9c relative to earth and the Klingon Bird-of-Prey moves at 0.7c relative to earth. What does the navigator of the Bird-of-Prey report for the speed of the Enterprise?
11. The Klingon Battle Cruiser moving at 0.7c relative to earth fires a torpedo at 0.5c relative to the Battle Cruiser. What is the speed of the torpedo as observed from earth?
12. The Klingon Battle Cruise approaches earth at 0.7c relative to earth. It passes the Ferengi Shuttle at 0.5c relative to the shuttle. What is the speed of the Shuttle relative to the earth?
13. The starship Enterprise approaches the planet Risa at a speed of 0.5c relative to the planet. On the way, it overtakes the intergalactic freighter Astra. The relative speed of the two ships as measured by the navigator on the Enterprise is 0.1c. If the Astra has a red (λ = 650 nm) navigation light, what wavelength will the Enterprise see as they approach the Astra.

3. $\Delta t = 24\ h, v = 1 \times 10^8 \frac{m}{s},\ \Delta t_0 = ?$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$24\ h = \frac{\Delta t_0}{\sqrt{1 - \frac{\left(1 \times 10^8 \frac{m}{s}\right)^2}{\left(3 \times 10^8 \frac{m}{s}\right)^2}}}$$

$$24\ h = \frac{\Delta t_0}{\sqrt{1 - \frac{1}{9}}}$$

$$24\ h \sqrt{\frac{8}{9}} = \Delta t_0$$

$$\boldsymbol{\Delta t_0 = 22.6\ h}$$

4. $v = 0.90c, \Delta t = 0.50\ s, \Delta t_0 = ?$

$$0.50\ s = \frac{\Delta t_0}{\sqrt{1 - \frac{(0.90c)^2}{c^2}}}$$

$$0.50\ s \sqrt{1 - 0.90^2} = \Delta t_0$$

$$\boldsymbol{\Delta t_0 = 0.218\ s}$$

5. $L_0 = 1\ m, L = 0.5\ m, v = ?$

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

$$0.5\ m = 1\ m \sqrt{1 - \frac{v^2}{c^2}}$$

$$0.25 = 1 - \frac{v^2}{c^2}$$

$$0.75 = \frac{v^2}{c^2}$$

$$v = \sqrt{0.75}c$$

$$\boldsymbol{v = 0.87c}$$

6. $v = 0.99c, m = 9.11 \times 10^{-31}\ kg, p = ?$

$$p = \frac{mv}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$p = \frac{\left(9.11 \times 10^{-31}\ kg\right)\left(0.99\left(3 \times 10^8 \frac{m}{s}\right)\right)}{\sqrt{1 - \frac{(0.99c)^2}{c^2}}}$$

$$\boldsymbol{p = 1.92 \times 10^{-21}\ kg\ m/s}$$

7. $m = 500\ kg, v = 0.90c, KE = ?$

$$KE = mc^2\left(\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} - 1\right)$$

$$KE = (500\ kg)(c^2)\left(\frac{1}{\sqrt{1 - \frac{0.90c^2}{c^2}}} - 1\right)$$

$$\boldsymbol{KE = 5.82 \times 10^{19}\ J}$$

8. $m = 2\ kg, E = ?$

$$E = mc^2$$

$$E = (2\ kg)\left(3 \times 10^8 \frac{m}{s}\right)^2$$

$$\boldsymbol{E = 1.8 \times 10^{17}\ J}$$

9. $KE_{classic} = \frac{1}{2}mv^2$

$$KE_{classic} = \frac{1}{2}(500\ kg)\left(0.90\left(3 \times 10^8 \frac{m}{s}\right)\right)^2$$

$$KE_{classic} = 1.82 \times 10^{19}\ J$$

$$KE_{relativistic} = 5.82 \times 10^{19}\ J\ \textit{(see #7)}$$

$$ratio = \frac{5.82 \times 10^{19}\ J}{1.82 \times 10^{19}\ J} = \boldsymbol{3.20}$$

10. $v_{EntE} = 0.9c, v_{KE} = 0.7c, v_{EntK} = ?$

$$v_{EntK} = \frac{v_{EntE} + v_{EK}}{1 + \frac{v_{EntE}v_{EK}}{c^2}}$$

$$v_{EntK} = \frac{0.9c + -0.7c}{1 + \frac{(0.9c)(-0.7c)}{c^2}}$$

$$\boldsymbol{v_{EntK} = 0.541c}$$

11. $v_{KE} = 0.7c, v_{TK} = 0.5c, v_{tE} = ?$

$$v_{TE} = \frac{v_{TK} + v_{KE}}{1 + \frac{v_{TK}v_{KE}}{c^2}}$$

$$v_{TE} = \frac{0.5c + 0.7c}{1 + \frac{(0.7c)(0.5c)}{c^2}}$$

$$\boldsymbol{v_{TE} = 0.889c}$$

12. $v_{KE} = 0.7c, v_{KS} = 0.5c, v_{SE} = ?$

$$v_{SE} = \frac{v_{SK} + v_{KE}}{1 + \frac{v_{SK}v_{KE}}{c^2}}$$

$$v_{SE} = \frac{-0.5c + 0.7c}{1 + \frac{(-0.5c)(0.7c)}{c^2}}$$

$$\boldsymbol{v_{SE} = 0.308c}$$

13. $u = -0.1c, \lambda_s = 650\ nm$

$$\lambda_{obs} = \lambda_s \sqrt{\frac{1 + \frac{u}{c}}{1 - \frac{u}{c}}}$$

$$\lambda_{obs} = (650\ nm) \sqrt{\frac{1 + \frac{-0.1c}{c}}{1 - \frac{-0.1c}{c}}} = \boldsymbol{588\ nm}$$